

On models of gauge field localization on a brane

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Abstract

We argue that any viable mechanism of gauge field localization should automatically imply charge universality on the brane. We study whether this condition is satisfied in the two known proposals aimed to localize vector field in flat bulk space. We construct a simple calculable model with confinement in the bulk and deconfinement on the brane, as in the Shifman–Dvali set up. We find that in our model the 4-dimensional Coulomb law is indeed reproduced on the brane due to the massless localized photon mode. The charge universality is enforced by the presence of “confining strings”. On the other hand, charge universality condition is not satisfied in another, brane-induced localization mechanism when the number of extra dimensions d is larger than two. We demonstrate that in the non-Abelian case the gauge fields inside the brane are never four-dimensional and their self-interaction is strong at all distances of interest. Hence this mechanism does not work for $d > 2$. At $d = 2$ the charge universality is still a problem, but it holds automatically at $d = 1$. At $d = 1$, however, the bulk gauge fields are strongly coupled in the non-Abelian case.

1 Introduction and Summary

Phenomenological discussions of various “brane world” scenarios are sometimes based on field theoretic models for localization of matter fields and possibly gravity on a three-dimensional submanifold — brane — embedded in higher dimensional space. A simple mechanism of the localization of fermions (and also scalars) makes use of domain walls or topological defects of higher codimension [1, 2]: in the presence of a topological defect, there often exist (chiral) fermion zero modes whose wave functions concentrate near the defect [3, 4, 5]. Somewhat similar mechanism localizes gravitons in the vicinity of a gravitating domain wall embedded in five-dimensional space

[6, 7]; higher-dimensional generalizations of the latter mechanism have been proposed in Refs. [8, 9, 10] (see, however, Ref. [11]). Gravity of a brane is capable of localizing scalars [12] and gauge fields [13, 14] as well (for a review see, e.g., Ref. [15]).

Some brane world scenarios, notably ADD [16], require non-gravitational mechanisms of matter localization. In this regard, it is of interest to understand possible ways of trapping *gauge fields* to a brane in flat multi-dimensional space. The simple zero mode mechanism, similar to that localizing fermions and scalars on a defect, is unlikely to work for gauge fields, for the following reason. Suppose that $(3 + 1)$ -dimensional fermions and gauge fields are zero modes of bulk spinors and vectors, respectively, *i.e.* they are described by the wave functions

$$\begin{aligned}\Psi(x^\mu, z^\alpha) &= \psi(x^\mu) \Psi_0(z^\alpha) \\ A_\mu(x^\nu, z^\alpha) &= A_\mu(x^\nu) \mathcal{A}(z^\alpha) ,\end{aligned}$$

where x^μ ($\mu = 0, 1, 2, 3$) and z^α ($\alpha = 1, \dots, d$) are coordinates along the brane and transverse to the brane, respectively, $\psi(x^\mu)$ and $A_\mu(x^\nu)$ are the usual four-dimensional wave functions (say, plane waves) and $\psi_0(z^\alpha)$ and $\mathcal{A}(z^\alpha)$ are the wave functions in the transverse dimensions. The latter are supposed to correspond to bound states (massless from the four-dimensional point of view), so that $\psi_0(z^\alpha)$ and $\mathcal{A}(z^\alpha)$ are peaked near the brane, $z^\alpha = 0$, and decrease towards $|z| \rightarrow \infty$. Multi-dimensional gauge interactions would then induce interactions between fermions and vectors residing on the brane, with the fermion gauge charges in the four-dimensional effective theory being proportional to the overlap integrals

$$\int d^d z \Psi_0^\dagger(z^\alpha) \mathcal{A}(z^\alpha) \Psi_0(z^\alpha) . \quad (1)$$

The problem is that these overlap integrals may take arbitrary values: the shapes of the fermion zero mode wave functions depend on the details of the interaction of bulk fermions to the defect and are different, at least in principle, for different fermionic species. Hence, the zero mode mechanism would allow for different (and arbitrary) values of gauge charges for different types of fermions, which should not be possible, at least in the non-Abelian case.

Any viable mechanism of gauge field localization should automatically lead to equal gauge charges of matter fields residing on the brane, *i.e.*, should

automatically ensure charge universality. This property is not inherent in the simple zero mode mechanism in flat background, so it has no chance to work¹.

In this paper we discuss two proposals for localizing gauge fields. One of them [17] is based on the assumption that the gauge theory is confining in the bulk, whereas confinement is absent on the brane². In Section 2 we substantiate this proposal by modelling this situation in a theory of dual superconductivity. For simplicity, we consider the case of one extra dimension; then this theory contains a two-form field with the mass parameter $\Lambda(z)$ depending on the extra coordinate z . For constant Λ this theory [18, 19] possesses the 't Hooft–Mandelstam mechanism of confinement. We will see that with $\Lambda(z)$ vanishing on the brane, charges residing on the brane experience four dimensional Coulomb law, and that the zero mode of the gauge field appears. We will also discuss how charge universality is enforced by confinement in the bulk.

Another proposal [20] is to modify the action of the gauge field by adding to conventional bulk action a term concentrating on a brane; this proposal is based on earlier discussion of the possibility to localize gravity on a brane in this way [21, 22]. In the limit of zero brane thickness, the additional term is

$$S_{brane} \propto \int d^4x \, d^d z \, \delta^d(z) \, F_{\mu\nu}^2(x, z) . \quad (2)$$

It has been argued that this term may be induced by loops involving particles residing on the brane, and that there is a wide range of distances at which the gauge theory on the brane is effectively four-dimensional [20].

The brane-induced localization mechanism works differently for one extra dimension, $d = 1$, and larger number of extra dimensions. At $d = 1$, the relevant gauge field propagator is non-singular on the brane, whereas at larger d it has a singularity at $z = 0$ (if the δ -function in eq. (2) is not regularized). An interpretation of the latter is that for $d > 1$, charges placed exactly on the brane and charges slightly displaced from the brane interact in a

¹It is worth commenting on how the charge universality obstruction is avoided in models where gauge fields are localized by gravity [13, 14]. In these models, the gauge field zero mode \mathcal{A} is *independent* of z , and yet it is normalizable with appropriate measure determined by the bulk geometry. The overlap integral analogous to (1) coincides then with the norm of Ψ_0 , so gauge charges are in fact universal.

²We note in passing that, to the best of our knowledge, a "microscopic" higher-dimensional quantum field theory possessing these properties, is yet unknown.

quite different way. This signalizes for the lack of charge universality, so one suspects that the brane-induced localization may have problems at $d > 1$.

In fact, the regularization of the δ -function in the action (2) is a necessity, otherwise there are uncontrollable singularities. The simplest regularization is smearing this δ -function out (cf. Ref. [20], another regularization is proposed in Ref. [23]). In Section 3 we adopt this approach and consider the case $d > 2$. We find that, in a sense, the gauge field inside the brane is never four-dimensional, which is unacceptable in non-Abelian theory. Furthermore, in the non-Abelian case the gauge coupling in the theory inside the brane is strong at all distances of interest. We conclude that this model does not admit semiclassical treatment, so the mechanism of Ref. [20], as it stands, does not work for non-Abelian gauge fields and $d > 2$.

While at $d = 2$ the charge universality is still a problem, the situation is different at $d = 1$. This case is also considered in Section 3. We find that in this case, the charge universality holds automatically in effective four-dimensional theory. In non-Abelian gauge theory, however, there is another problem: if one requires the effective four-dimensional gauge coupling to be roughly of order one, the original five-dimensional theory is strongly coupled in the bulk at distance scales of interest.

To conclude this section, let us point out that a gravitational analogue of the charge universality is the equivalence principle. So, it is natural to conjecture that any self-consistent theory (quasi-)localizing graviton on a brane should automatically lead to the four-dimensional equivalence principle. This property is indeed present in the Randall–Sundrum proposal [7], but does not appear to be inherent in the brane-induced localization mechanism of Refs. [21, 22]. We point out in Section 3 that gravity “localized” by the brane-induced mechanism has the same problems at $d > 2$ as gauge fields, whereas at $d = 1$ the four-dimensional equivalence principle is ensured. Hence, if it was not for the problem with the tensor structure of the graviton propagator [21] and the mismatch of scales (see Ref. [23] and subsection 3.2), the brane-induced mechanism would be a viable scenario of the localization of gravity in *five* flat dimensions.

2 Confinement in the bulk, no confinement on the brane

2.1 Model for dual superconductivity

Let us consider space-time of D “our” dimensions ($D \geq 3$) and one extra dimension. To construct an explicit model with confinement in the bulk and no confinement on the brane, we begin with the theory [18] exhibiting the ’t Hooft–Mandelstam mechanism of confinement. For the time being, we consider homogeneous $(D + 1)$ -dimensional space-time; we will introduce the brane later on. Microscopically, one thinks of an Abelian theory with monopoles, and assumes that there is a phase with monopole condensate. Then electric charges are confined. Phenomenologically, this situation may be described by the low energy effective theory involving a two-form field ω_{ab} ($a, b = 1, \dots, D + 1$) which generalizes the Maxwell field strength. The sources are represented by an anti-symmetric tensor T_{ab} related to the usual electromagnetic current as follows

$$\partial_b T^{ab} = \frac{1}{2} j^a . \quad (3)$$

The action for this model is

$$S = \int d^{D+1} X \left(\frac{1}{12\Lambda^2} \Omega_{abc} \Omega^{abc} + \frac{1}{4e^2} \omega_{ab} \omega^{ab} + i\omega_{ab} T^{ab} \right) , \quad (4)$$

where Ω_{abc} is the field strength,

$$\Omega_{abc} = \partial_a \omega_{bc} + \partial_c \omega_{ab} + \partial_b \omega_{ca} . \quad (5)$$

We work in Euclidean $(D + 1)$ -dimensional space-time, hence the factor i in the action (4). The parameter Λ may be interpreted as the energy scale of the monopole condensate, and e is the electric charge. It is implicit in (4) that the theory has a fixed ultraviolet cut-off μ ; we will see shortly that the tension of the string between electric charges depends on this cut-off.

As explained in Ref. [18], this action can be obtained by a certain gauge fixing from a more general Abelian action. At $D = 3$ the latter action is dual to the Abelian Higgs model with frozen vacuum expectation value of the Higgs field. The two-form field ω_{ab} is dual to the phase of the Higgs field.

At $D > 3$ the action before gauge fixing is dual to the generalized Higgs model describing $(D - 2)$ -form field dual to the electromagnetic field, and $(D - 3)$ -form field dual to the antisymmetric tensor ω_{ab} . This $(D - 3)$ -form is a generalization of the phase of Higgs field in the four-dimensional Abelian Higgs model.

It is straightforward to see that when the monopole condensate vanishes, $\Lambda \rightarrow 0$, the model (4) reduces to the usual QED. Indeed, finiteness of the action implies that in this limit

$$\Omega_{abc} = 0 \quad (6)$$

and, consequently,

$$\omega_{ab} = \partial_a A_b - \partial_b A_a \quad (7)$$

with some vector field A_a . Then the relation (3) implies that the last two terms in Eq. (4) describe photon field A_a interacting with electromagnetic current j_a .

For non-vanishing Λ the situation is not so simple. In the case of two opposite point charges, Eq. (3) implies that

$$T^{ab} = -\frac{1}{2} \int d^2\sigma \epsilon^{\alpha\beta} \frac{\partial x^a}{\partial \sigma^\alpha} \frac{\partial x^b}{\partial \sigma^\beta} \delta^{D+1}(x - x(\sigma)) \quad (8)$$

where $x(\sigma)$ parametrizes a surface Σ bounded by the world-lines of the charges. Hence the action (4) as it stands depends on the choice of this surface. The potential between electric charges in dual superconductor should, however, depend only on the locations of these charges. The right way to ensure the latter property is to integrate over all surfaces Σ [19]. This integration is interpreted as the integration over world sheets of confining strings. We will perform this integration semiclassically, taking the surface which minimizes the action. In most cases considered below this surface is uniquely determined by symmetries.

The solution to the classical field equations following from the action (4) is

$$\omega_{ab} = -i \frac{2\Lambda^2}{m^2 - \Delta_{D+1}} T_{ab} - i \frac{e^2}{m^2 - \Delta_{D+1}} (\partial_a j_b - \partial_b j_a), \quad (9)$$

where Δ_{D+1} is $(D + 1)$ -dimensional Euclidian Laplacian, and mass m is equal to

$$m = \frac{\Lambda}{e}.$$

It is straightforward to check that an analogue of the first pair of Maxwell's equations is satisfied,

$$\partial_a \omega_{ab} = ie^2 j_b . \quad (10)$$

This is in accord with the interpretation of ω_{ab} as the generalization of the Maxwell tensor in the phase with monopole condensate. Substituting the field (9) back into the action (4) and integrating by parts one obtains the following expression for the action describing the interaction of electric charges,

$$S = \int d^{D+1}x \left(T_{ab} \frac{\Lambda^2}{m^2 - \Delta_{D+1}} T_{ab} + \frac{e^2}{2} j_a \frac{1}{m^2 - \Delta_{D+1}} j_a \right) . \quad (11)$$

The second term in Eq. (11) is local and does not depend on the choice of the string world sheet. It reduces to the usual Coulomb term at small charge separation, $L \ll m^{-1}$ and exponentially decreases at larger distances. The first term in Eq. (11) is non-local and gives rise to the confining potential between charges.

To see the property of confinement explicitly, let us consider two static charges located at the origin and at the point $X_1 = L$, $X_2 = \dots = X_D = 0$. In this case the minimal surface is clearly a flat rectangular, infinite in time direction. The non-vanishing components of T_{ab} are

$$T_{0l} = -T_{l0} = \delta^{D-1}(X_l) \theta(X_1) \theta(L - X_1) \quad (12)$$

where l runs from 2 to D . In this static case, the action (11) determines the potential between the charges,

$$S = V(L)T .$$

The non-local term in Eq. (12) gives the following contribution to the potential (to the leading order in m/μ)

$$V_{conf}(L) = \sigma L , \quad (13)$$

where the string tension σ explicitly depends³ on the cutoff μ ,

$$\sigma \propto \Lambda^2 \mu^{D-3} . \quad (14)$$

³Explicit dependence of the string tension σ on the cutoff scale μ is due to the divergence of the integral over the coordinates transverse to the string worldsheet. Equation (14) is in agreement with the well-known logarithmic divergence in the energy of the usual ($D = 3$) Abrikosov–Nielsen–Olesen vortex in the limit of infinite Higgs mass.

The Coulomb and confinement regimes occur at short and long distances, the transition between the two takes place at the scale where confining and Coulomb potentials are of the same order,

$$L_c \sim \left(\frac{e^2}{\sigma}\right)^{\frac{1}{D-1}} \sim \left(\frac{e^2}{\Lambda^2 \mu^{D-3}}\right)^{\frac{1}{D-1}}. \quad (15)$$

Note that $L_c \ll m^{-1}$ so that in the whole region where the local contribution dominates, the potential between the charges is indeed of the Coulomb type.

2.2 Potential between charges on the brane

To consider the situation proposed by Dvali and Shifman as a mechanism of the localization of gauge fields [17], we modify the above model and take the scale of confinement Λ to be a non-trivial function in transverse space, which is supposed to vanish on the brane surface. For the sake of simplicity we consider flat co-dimension one brane, *i.e.* assume that $\Lambda = \Lambda(z)$ is a function of one coordinate $z \equiv X^{D+1}$. The brane is placed at $z = 0$, and symmetry $z \rightarrow -z$ is assumed. In order to avoid singularities in the action (4) we work with small, but non-vanishing $\Lambda(0)$. The corresponding mass $m_0 = \Lambda(0)/e$ is the smallest energy scale in the problem. We will not specify the explicit form of the function $\Lambda(z)$ for the moment and require only that outside the region of small size z_0 , the confinement scale $\Lambda(z)$ is constant⁴ and large enough, $\Lambda(|z| > z_0) \equiv \Lambda_c \gg \Lambda(0)$.

The field equation following from the action (4) with varying Λ has the form

$$\partial_a (1/\Lambda^2(z) \Omega_{abc}) - \frac{1}{e^2} \omega_{bc} - 2iT_{bc} = 0 \quad (16)$$

Taking the divergence of this equation we again obtain the first pair of Maxwell's equations, Eq. (10). It is straightforward to solve Eq. (16) for a general source T_{ab} . The result is

$$\omega_{z\mu} = \frac{-i}{\mathcal{D}_T - \Delta_D} [\Lambda^2(z) T_{z\mu} + e^2 (\partial_z j_\mu - \partial_\mu j_z)] \quad (17)$$

⁴Our analysis remains valid for $\Lambda(z)$ growing towards $|z| \rightarrow \infty$. In fact, this is the case in a specific example to be considered later, see Eq. (34).

and

$$\omega_{\mu\nu} = \frac{-i}{\mathcal{D}_L - \Delta_D} \left[\Lambda^2(z) T_{\mu\nu} + e^2 (\partial_\mu j_\nu - \partial_\nu j_\mu) - 2i \frac{\Lambda'(z)}{\Lambda(z)} (\partial_\mu \omega_{z\nu} - \partial_\nu \omega_{z\mu}) \right] \quad (18)$$

where transverse and longitudinal operators \mathcal{D}_T and \mathcal{D}_L are

$$\mathcal{D}_T = m^2(z) - \partial_z^2 \quad (19)$$

and

$$\mathcal{D}_L = m^2(z) - \partial_z^2 + 2 \frac{\Lambda'(z)}{\Lambda(z)} \partial_z \quad (20)$$

Indices μ, ν run from 0 to D , and prime denotes differentiation with respect to z .

Let us consider two static point-like charges located on the brane at distance L from each other. According to the qualitative arguments due to Dvali and Shifman, the D -dimensional (rather than $(D+1)$ -dimensional) Coulomb potential should emerge at large distances in this case. Since we keep $\Lambda(0)$ finite, charges on the brane experience confinement at very large distances, so this picture does not hold beyond the confinement scale on the brane

$$L_c(0) = \left(\frac{e^2}{\Lambda^2(0) \mu^{D-3}} \right)^{\frac{1}{D-1}}. \quad (21)$$

We will consider the distances $L \ll L_c(0)$, but still larger than all other scales inherent in the model. Our purpose is to see whether the D -dimensional Coulomb potential between charges on the brane indeed emerges at these distances.

From the symmetry $z \rightarrow -z$ and from the fact that the confinement scale $\Lambda(z)$ is minimal at $z = 0$ it is clear that the minimal surface in our case is the same as for non-vanishing constant Λ , so the source T_{ab} is again given by Eq. (12). Then, as in the usual electrodynamics, only electric field is non-vanishing. From Eqs. (17) and (18) we find

$$E_z \equiv \frac{1}{i} \omega_{z0} = -e^2 (\mathcal{D}_T - \Delta_D)^{-1} \partial_z j_0 \quad (22)$$

for the component of the electric field transverse to the brane, and

$$E_i \equiv \frac{1}{i} \omega_{i0} = -(\mathcal{D}_L - \Delta_D)^{-1} \left[\Lambda^2(0) T_{i0} + e^2 \partial_i j_0 + 2 \frac{\Lambda'(z)}{\Lambda(z)} \partial_i E_z \right], \quad (23)$$

for the electric field parallel to the brane ($i = 1, \dots, (D - 1)$). Equation (10) becomes the $(D + 1)$ -dimensional Gauss' law,

$$\partial_z E_z + \partial_i E_i = e^2 j_0 \quad (24)$$

The operator (19) entering the expression (22) for the transverse electric field is a Schrödinger operator with the potential $m^2(z)$ having the shape of a well of characteristic width z_0 and characteristic height $(\Lambda_c/e)^2$. The mass of the lightest mode corresponding to this operator is of order

$$m_T \sim \min\{z_0^{-1}, \Lambda_c/e\} \quad (25)$$

For small enough $\Lambda(0)$, this mass is much larger than $L_c(0)^{-1}$. Then a range of intermediate distances L exists, where E_z component of the electric field is already exponentially small but confinement on the brane has not yet set in. We are interested precisely in this range of distances between the charges.

At these distances the stringy term, $\Lambda^2(0)T_{i0}$, in the expression (23) for the longitudinal part of the electric field is negligible. The remaining terms in Eq. (23) are local, so it makes sense to consider electric field of *one* charge. From Eqs. (22) and (23) it is clear that this field is symmetric under the spatial rotations on the brane. The field E_z vanishes as $|z| \rightarrow \infty$, so eq. (24) in its integral form becomes the D -dimensional Gauss' law,

$$\int d\sigma^i E_i^{(l)} = e^2 q \quad (26)$$

for the integrated longitudinal field

$$E_i^{(l)} = \int_{-\infty}^{+\infty} dz E_i(z, \mathbf{x})$$

In eq. (26), integration is performed over a $(D - 2)$ -dimensional sphere of radius L . Now, the longitudinal field also vanishes at large $|z|$, so eq. (26) implies

$$E^{(l)} \propto \frac{1}{L^{D-2}}, \quad (27)$$

This suggests that the interaction strength between the two charges indeed follows the D -dimensional Coulomb law.

2.3 Zero mode

In order to substantiate the above semi-quantitative argument let us study the spectrum of the longitudinal operator (20). This operator is Hermitean with the measure $\Lambda^{-2}(z)$, and is positive definite. The existence of the long-range field (27) implies that the operator (20) has nearly zero mode with mass

$$m_{light} \lesssim L_c^{-1}(0). \quad (28)$$

To see this explicitly let us make use of the inequality

$$\int \frac{dz}{\Lambda^2(z)} \psi^* \mathcal{D}_L \psi > m_{light}^2 \int \frac{dz}{\Lambda^2(z)} \psi^* \psi, \quad (29)$$

where the weight is chosen according to the Hermiticity property of \mathcal{D}_L . Here $\psi(z)$ is an arbitrary continuous function vanishing at infinity. Let us consider the trial function ψ of the following form

$$\psi(z) = \exp \left(-\frac{1}{e} \int_0^z dz \bar{\Lambda}(z) \right), \quad (30)$$

where the function $\bar{\Lambda}(z)$ is defined as follows,

$$\bar{\Lambda}(z) = \text{sgn}(z) \Lambda(z). \quad (31)$$

With this trial function, the estimate for the mass of the lightest mode is

$$m_{light}^2 \sim \frac{2 \int \psi^2 dz}{e^2 \int \frac{dz}{\Lambda^2(z)} \psi^2}. \quad (32)$$

Now it is clear, that if $\Lambda(0)$ tends to zero, then the mass of the lightest mode also vanishes. If one introduces parameters z_0 and k^{-1} which determine the widths of the regions near $z = 0$ where $\psi(z) \sim 1$ and $\Lambda(z) \sim \Lambda(0)$, respectively, then from (32) one has the following estimate (assuming $k > z_0^{-1}$)

$$m_{light}^2 \sim \frac{1}{e^2} \Lambda^2(0) k z_0 \quad (33)$$

from which it follows that the relation (28) is valid.

To illustrate the above general reasoning let us consider a specific choice of $\Lambda(z)$ in more detail. Namely, we chose $\Lambda(z)$ in the form

$$\Lambda(z) = \Lambda_c e^{k(|z|-z_0)} . \quad (34)$$

We will assume the following relation between various dimensionful parameters,

$$\Lambda(0) = \Lambda_c e^{-kz_0} \ll \left(z_0^{-1}, \frac{\Lambda_c}{e} \right) \ll k \ll \mu \quad (35)$$

To take the limit of small $\Lambda(0)$, we keep Λ_c and z_0 fixed, and take k large. Then z_0 and k are essentially the same parameters that enter our qualitative estimate, eq. (33). The eigenvalue equation for the operator (20) has the following form at $z > 0$,

$$-\psi'' + 2k\psi' + \left(\frac{\Lambda_c}{e} \right)^2 e^{2k(z-z_0)} \psi = p^2 \psi \quad (36)$$

The general bounded at infinity solution to Eq. (36) is

$$\psi(z) = N e^{kz} K_\nu \left(\frac{\Lambda_c}{ek} e^{k(z-z_0)} \right) , \quad (37)$$

where N is a normalization factor and the order ν of the modified Bessel function K_ν is equal to

$$\nu = \sqrt{1 - \frac{p^2}{k^2}}$$

Let us first consider symmetric eigenfunctions. They obey

$$\psi'(0) = 0 . \quad (38)$$

which gives

$$(1 + \nu)K_\nu(\xi_0) - \xi_0 K_{\nu+1}(\xi_0) = 0 , \quad (39)$$

with

$$\xi_0 = \frac{\Lambda_c}{ek} e^{-kz_0} = \frac{\Lambda(0)}{ek} \ll 1 . \quad (40)$$

Equation (39) determines the eigenvalues p^2 . The lowest one is

$$m_{light}^2 = 2 \left(\frac{\Lambda(0)}{e} \right)^2 \left(kz_0 - \ln \left(\frac{\Lambda_c}{ek} \right) \right) , \quad (41)$$

whereas higher eigenvalues are separated by the gap of order k (in fact, in the limit $\xi_0 \rightarrow 0$, the next-to-lowest eigenvalue tends precisely to k). The inequality (28) is satisfied and the light mode behaves as massless at distances of interest, $L < L_c(0)$. Clearly, in the limit $\Lambda(0) \rightarrow 0$, the light mode becomes exactly massless. It is this massless mode that mediates D -dimensional Coulomb interaction between charges on the brane. Note that the result (41) is in agreement with our previous estimate (33).

The anti-symmetric eigenfunctions obey

$$\psi(0) = 0$$

According to eq. (37), this is possible only for $p > k$, when ν is imaginary. So, all eigenvalues but one are large, and the light mode is the only one relevant at large distances.

2.4 Charge universality

Let us now discuss the charge universality in this set up. To this end, let us consider two point-like opposite charges displaced from the brane to the points z_+ and z_- , respectively. Clearly, the general case of a continuous charge distribution in z -direction can be straightforwardly obtained from this one.

Naively, one may expect that the following two effects take place. First, the confinement length between two charges may appear to be determined now not by $\Lambda(0)$ as in Eq. (21), but by $\Lambda(z_c)$ where z_c takes some intermediate value between z_+ and z_- . Second, in the distance interval where confinement between the two charges has not set in yet, the long range force is transmitted by the light mode. Then the general argument, described in Introduction, suggests that this force depends on the overlaps between the charge distributions and wave function of the light mode. In other words, one might expect that the force between the two charges is proportional to $\psi_0(z_+) \cdot \psi_0(z_-)$, where $\psi_0(z)$ is the light mode. This would mean the lack of charge universality. However, we will see that neither of these two effects actually occurs.

As before, let us take the distance between the charges along the brane, L , to be much smaller than the confinement length on the brane, $L_c(0)$. The key point is that outside an L -independent neighborhood of each of the charges,

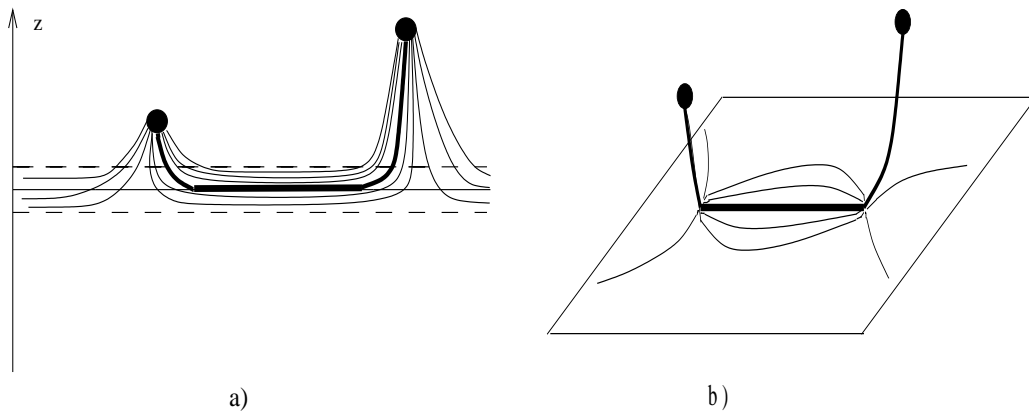


Figure 1: A string (thick line) and electric field lines for two charges placed in the bulk: a) side view; b) overall picture (electric field lines are shown on the brane only).

the minimal string *lays on the brane*: clearly, any other string configuration has higher energy and action. This configuraion is illustrated in Figure 1, where the electric field lines are also shown.

Then, as in the case of point charges located on the brane, there are two different components of the electric field. The first one is non-local and originates from the long string running along the brane. It is this component that is responsible for linear potential between the charges, so confinement scale is $L_c(0)$, as before. The second component is (almost) local and comes from the regions near the charges; it includes, in particular, the contributions of the parts of the string which connect the charges to the brane. The non-local component is suppressed by the small value of the confinement scale on the brane, $\Lambda(0)$ (cf. Eqs. (17) and (18)), and is negligible at the distances between charges smaller than $L_c(0)$, which is the case we consider. For the local component it makes sense to consider electric fields of each of the charges separately. Sufficiently far away from the charges, the long range component of this field is due to the light mode and has the form

$$E_i = \hat{\mathbf{x}}_i \frac{a}{\Omega_{D-2}} \frac{\psi_0(z)}{L^{D-2}}, \quad (42)$$

where L is the distance to the charge, Ω_{D-2} is the area of unit $(D-2)$ -dimensional sphere and $\hat{\mathbf{x}}$ is unit radius-vector. The coefficient a is determined by the Gauss' law (26),

$$a = \frac{qe^2}{\int dz \psi_0(z)}. \quad (43)$$

We see that this coefficient is independent of the position of the charge in transverse direction, and is determined by the value of charge q only. Hence, charge universality holds in our model.

To find the effective charge q_{eff} in the low-energy effective theory on the brane, let us calculate the interaction energy of the two charges. As in the usual electrodynamics, the Gaussian structure of the action (4) implies that this energy may be calculated as an energy of the two-form field $\omega_{0i} = iE_i$ produced by the charges. The contribution into the energy integral that dominates at large L , comes from the region where both $|\mathbf{x} - \mathbf{x}_+|$ and $|\mathbf{x} - \mathbf{x}_-|$ are of order L , where \mathbf{x}_\pm are the longitudinal coordinates of the charges. The regions near the charges contribute into higher multipoles only⁵.

Using Eq. (42), one obtains for the interaction energy

$$V(L) = -\frac{a^2}{4\Omega_{D-2}^2} \left(\int \frac{d^{D-1}\mathbf{x}}{|\mathbf{x} - \mathbf{x}_+|^{D-2} |\mathbf{x} - \mathbf{x}_-|^{D-2}} \right) \int dz \left[\frac{1}{12\Lambda^2} (\partial_z \psi_0)^2 + \frac{1}{4e^2} \psi_0^2 \right].$$

The integral over longitudinal coordinates \mathbf{x} gives the D -dimensional Coulomb behavior, $V(L) \propto 1/L^{D-3}$, precisely in the same way as in the usual electrodynamics. Evaluating the integral over z by parts and using the fact that $\psi_0(z)$ is an eigenmode of the operator \mathcal{D}_L with the eigenvalue m_{light}^2 we obtain

$$q_{eff}^2 = a^2 m_{light}^2 \int dz \frac{\psi_0^2(z)}{\Lambda^2(z)}$$

or, making use of Eq. (43),

$$q_{eff}^2 = q^2 e^4 m_{light}^2 \frac{\int dz \frac{\psi_0^2(z)}{\Lambda(z)^2}}{\left(\int dz \psi_0(z) \right)^2}.$$

Now, recalling the estimate (32) for m_{light} we find that q_{eff} remains finite in the limit $\Lambda(0) \rightarrow 0$. Under the same conditions that lead to eq.(33), the estimate for the effective charge is

$$q_{eff}^2 \sim \frac{q^2}{z_0}.$$

To summarize, in the low energy effective theory, the charges are finite and universal.

⁵It is interesting to note that higher multipoles are *not* universal, as they depend on z_\pm .

2.5 Discussion

In our model, the d -dimensional Coulomb law between charges placed on or near the brane comes together with the zero mode of the gauge field. This is in accord with the general rule of the existence of a massless particle, photon, travelling along the brane, in theories exhibiting the Coulomb law on the brane. Another important ingredient are thin strings between the charges. These strings are invisible in the low energy effective theory (for $\Lambda(0) \rightarrow 0$), as they lay on the brane, but play a key role in ensuring charge universality. For charges displaced from the brane (or non-trivial charge distributions in the transverse direction), the short string connecting the charge to the brane should contribute to the self-energy of the charge. There may be other low-energy effects of these short strings, such as polarizability of charged particles whose wave functions have finite spread in the transverse direction.

The model discussed in this section has exotic modifications. For instance one may consider several branes parallel to each other as in Ref. [24]. Another modification with potentially interesting phenomenological consequences is as follows. Suppose that the confinement scale on our brane is finite, as it happens in QCD, and that $\Lambda(z)$ has a local minimum on our brane. Suppose further that $\Lambda(z)$ has a global minimum on some other brane or in the bulk, and that $\Lambda = 0$ there. Then there should exist "free quarks" whose electric field is shown in Figure 2. These objects will be colorless, but otherwise have quantum numbers of a quark. Their masses will be determined by the properties of the flux tube, extending into extra dimensions and connecting the free quarks to the region with $\Lambda = 0$; this mass may naturally be quite large.

3 Brane-induced localization

Let us now turn to another proposal [20]. Consider $(4 + d)$ -dimensional flat space-time with coordinates x^μ ($\mu = 0, 1, 2, 3$) and z^α ($\alpha = 1, \dots, d$), and assume that there is a brane at $z^\alpha = 0$. It has been proposed to choose the action for the gauge field in the following form,

$$S = \frac{1}{g_{(4+d)}^2} \int d^4x d^d z \left(\frac{1}{4} F_{ab}^2 + \frac{1}{4} C^2 \delta^d(z) F_{\mu\nu}^2 \right) \quad (44)$$

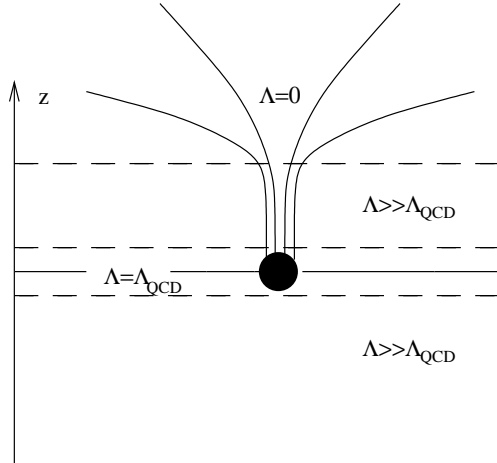


Figure 2: Color-electric field lines of a “free quark”

where F_{ab} is the $(4+d)$ -dimensional field strength, $F_{\mu\nu}$ are its four-dimensional components, C is some constant of dimension $M^{-d/2}$; we work in Euclidean space-time for convenience. The second term in eq. (44) concentrates on the brane; the discussion of how the term of this type may be generated is given in Refs. [21, 22, 20].

3.1 More than two extra dimensions

To be specific, let us first consider the case $d > 2$. A heuristic argument suggesting that charges residing on the brane experience the four-dimensional Coulomb law is as follows (cf. Ref. [22]). Omitting indices, and ignoring complications due to gauge fixing, one writes the equation for the propagator,

$$(\square^{(4+d)} + C^2 \delta^d(z) \square^{(4)}) G(x, x'; z, z') = -\delta^4(x - x') \delta^d(z - z') \quad (45)$$

Placing the source on the brane, *i.e.* setting $z' = 0$, and using the four-dimensional momentum representation, one has

$$(\square^{(d)} - p^2 - C^2 p^2 \delta^d(z)) G(p; z) = -\delta^d(z) \quad (46)$$

where p_μ is the four-momentum. A formal solution to this equation is

$$G(p; z) = \frac{D(p; z)}{1 + C^2 p^2 D(p; 0)} \quad (47)$$

where $D(p; z)$ is the free $(4 + d)$ -dimensional propagator in a space-time without brane, which obeys

$$(\square^{(d)} - p^2) D(p; z) = -\delta^d(z) \quad (48)$$

Now, at $d > 2$, the free propagator $D(p; z)$ is finite at finite z^α and diverges as $z^\alpha \rightarrow 0$. So, one argues that $G(p; z) = 0$ at $z^\alpha \neq 0$, and

$$G(p; 0) = \frac{1}{C^2 p^2} \quad \text{at } z^\alpha = 0 \quad (49)$$

This quantity is proportional to the four-dimensional propagator, so one argues that the charges on the brane experience the four-dimensional Coulomb law. The effective four-dimensional gauge coupling would then be equal to

$$g_{(4)} = \frac{g_{(4+d)}}{C} \quad (50)$$

where $g_{(4+d)}$ is the gauge coupling in the original theory.

Of course, this argument is far from being rigorous [23]. The right hand side of eq. (47) does not exist (even in distributional sense), as $D(p; 0)$ is infinite. The δ -function in the action (44) has to be regularized. One way to do this is to smear this δ -function over a spherical region of small size z_0 ; it is precisely this regularization that we use in this paper.

As pointed out in Introduction, it is natural to suspect that this proposal has problems with charge universality. The above arguments imply that only that part of the charge which is contained in the region $|z| < z_0$ will participate in the four-dimensional Coulomb law (hereafter $|z| \equiv \sqrt{z^\alpha z^\alpha}$). The zero modes of matter fields may, however, spread over larger distances in transverse dimensions, so the four-dimensional charges of matter fields may depend on the shapes of their zero modes.

To see what the above set up actually corresponds to, let us consider the action

$$S = \frac{1}{g_{(4+d)}^2} \int d^4x d^d z \left(\frac{1}{4} F_{ab}^2 + \frac{1}{4} f^2(z) F_{\mu\nu}^2 \right) \quad (51)$$

where $f(z)$ is a step function equal to a constant f_0 at $|z| < z_0$ and zero at $|z| > z_0$. Omitting factors of order unity, we have a relation between parameters entering eqs. (44) and (51),

$$z_0^{\frac{d}{2}} f_0 \sim C \quad (52)$$

Clearly, f_0 is large for small z_0 .

Let us assume for simplicity that matter currents have components, transverse to the brane, equal to zero. In this case we can set the transverse components of the gauge field equal to zero, $A_\alpha = 0$, and write the action explicitly as follows,

$$S = \frac{1}{g_{(4+d)}^2} \int d^4x d^d z \left[\frac{1}{2} (\partial_{z^\alpha} A_\mu)^2 + \frac{1}{4} \hat{f}^2(z) F_{\mu\nu}^2 \right] \quad (53)$$

where $\hat{f}^2(z) = 1 + f^2(z)$. Let us consider the region $|z| < z_0$, in which $\hat{f}^2 = \text{const} = 1 + f^2 \equiv \hat{f}_0^2$. Since $f_0 \gg 1$, we will not distinguish between the constants \hat{f}_0 and f_0 in what follows. The propagator in this region is⁶ (again omitting indices)

$$G(x, x'; z, z') = \int \frac{d^4p d^d q}{(2\pi)^{4+d}} \frac{e^{iq(z-z')} e^{ip(x-x')}}{q^2 + f_0^2 p^2} \quad (54)$$

Now, the typical transverse momenta are of order $q \sim 1/z_0$, whereas the four-dimensional momenta are of order $p \sim 1/r$ where r is the distance along the brane. So, there is a characteristic distance scale

$$r_c = f_0 z_0 \sim \frac{C}{z_0^{\frac{d}{2}-1}} \quad (55)$$

where we made use of eq. (52). This scale is large at small z_0 (recall that we consider the case $d > 2$). Below this scale, the transverse momentum q in the denominator in eq. (54) is negligible, and the propagator is

$$G(x, x'; z, z') = \frac{1}{f_0^2} D^{(4)}(x - x') \delta(z - z') \quad (56)$$

⁶In fact, the correct expression for the propagator should be obtained by finding the solutions inside ($|z| < z_0$) and outside ($|z| > z_0$) the brane, and matching them at $|z| = z_0$. This leads to corrections to the expression (54). For sources which are spherically symmetric in transverse dimensions, the propagator can be found in explicit form. In this way one can show that corrections to eq. (54) are negligible at distances of interest.

where $D^{(4)}$ is the conventional massless four-dimensional propagator. This is how the expectation concerning the four-dimensional Coulomb law [22, 20] is confirmed. The propagator has four-dimensional behaviour at $r \ll r_c$, while extra dimensions "open up" at $r \sim r_c$.

To see how the relation (50) emerges, let us consider two static charge distributions $\rho(z)$ and $\rho'(z)$ spreading over the region $|z| < z_0$ and separated by a distance $r \ll r_c$ along the brane. According to eq. (56), the potential between them is

$$V(r) = \frac{g_{(4+d)}^2}{4\pi r} \frac{1}{f_0^2} \int d^d z \rho(z) \rho'(z) \quad (57)$$

Now, $\rho \sim q/z_0^d$, $\rho' \sim q'/z_0^d$, where q and q' are the total charges, so

$$V(r) \sim \frac{g_{(4+d)}^2}{f_0^2 z_0^d} \frac{qq'}{4\pi r} \quad (58)$$

Taking into account eq. (52), one indeed obtains the relation (50).

The result (57) is alarming, however. Not only the charge universality does not hold, but also the interaction between the sources is *ultra-local* in z . This calls for further analysis of this model.

Let us again consider the action (53) inside the region $|z| < z_0$ and change variables from z^α to

$$y^\alpha = f_0 z^\alpha$$

In terms of these variables, the action (53) becomes

$$S = \frac{1}{g_{(4+d)}^2 f_0^{d-2}} \int d^4 x d^d y [(\partial_{y^\alpha} A_\mu)^2 + F_{\mu\nu}^2] \quad (59)$$

This is the standard $(4+d)$ -dimensional action for (μ -components of) the gauge field. Now it is clear why the interaction (57) is ultra-local: in terms of the new variables, the charge distributions are huge pancakes of the transverse size

$$y_0 = f_0 z_0 \sim r_c$$

whereas the gauge fields propagate with the speed of light in all directions. At $r \ll r_c$, i.e., when the distance between the charged pancakes is much smaller than their size, the interaction occurs between the pieces of pancakes sitting in front of each other, hence the ultra-locality.

From the point of view of gauge fields themselves, space-time inside the brane, $|y| < y_0$, is $(4 + d)$ -dimensional, flat and has the transverse size of order r_c . This is unacceptable, at least in the non-Abelian case. In the first place, there are many more degrees of freedom than in the four-dimensional theory. Furthermore, according to eq. (59), the effective $(4 + d)$ -dimensional self-coupling of the gauge fields is

$$g_{(4+d)}^{eff} = g_{(4+d)} f_0^{\frac{d}{2}-1} \sim g_{(4)} r_c^{\frac{d}{2}} \quad (60)$$

where we made use of eqs. (52) and (55). Thus, for $g_{(4)}$ roughly of the order of unity, the gauge theory inside the brane becomes strongly coupled at distances (in all directions) just below r_c . This situation cannot be treated semiclassically, contrary to what has been implicitly assumed throughout the whole discussion.

The discussion of this subsection applies, word for word, to brane-localized gravity in more than two extra dimensions, with the obvious substitution

$$\begin{aligned} g_{(4+d)}^2 &\rightarrow L_{(4+d)}^{2+d} \equiv \frac{1}{M^{2+d}} \\ g_{(4)}^2 &\rightarrow L_{Pl}^2 \equiv \frac{1}{M_{Pl}^2} \end{aligned} \quad (61)$$

where M is the gravity scale in the underlying $(4 + d)$ -dimensional theory and M_{Pl} is the four-dimensional Planck mass. Gravity inside the brane ceases to be four-dimensional, and becomes strong at distances below the length scale obtained from eq. (60),

$$L_{(4+d)}^{eff} = (L_{Pl}^2 r_c^d)^{\frac{1}{2+d}}$$

For $d = 3$ and $r_c \sim 10$ kpc, this effective scale is of order 1 cm, which is unacceptably large. The scale $L_{(4+d)}^{eff}$ is even larger at $d > 3$.

3.2 One extra dimension

The cases $d = 1$ and $d = 2$ are special. Brane-induced localization in two extra dimensions has similar problems as in the case $d > 2$: at $d = 2$, the free propagator $D(p; z)$ is again singular at $z = 0$, so the discussion of the beginning of the previous subsection applies to the case $d = 2$ as well. The case $d = 1$ is different. Let us consider the latter case in some detail.

Let us assume for simplicity that charge distributions are symmetric under $z \rightarrow -z$, and consider the symmetric part of the propagator $G(x, x'; z, z')$. At $z, z' > 0$, the solution to eq. (45) with $d = 1$ is (in four-dimensional momentum representation)

$$G(p; z, z') = \frac{1}{2(r_c p^2 + p)} \left[r_c p \left(e^{-p|z-z'|} - e^{-p(z+z')} \right) + \left(e^{-p|z-z'|} + e^{-p(z+z')} \right) \right] \quad (62)$$

where

$$r_c \equiv C^2 = \frac{g_{(5)}^2}{g_{(4)}^2}$$

This propagator is finite at $z = 0$ and/or $z' = 0$, so one does not need to regularize the δ -function in the action (44). For sources on the brane, $z = z' = 0$, the propagator (62) agrees with the expression given in Ref. [21],

$$G(p; 0) = \frac{1}{r_c p^2 + p} \quad (63)$$

Charges placed on the brane at distance $r \ll r_c$ apart, experience the four-dimensional Coulomb law.

Charge distributions of width z_0 in transverse direction also experience the four-dimensional Coulomb law, provided the first term in square brackets in eq. (62) is small compared to the second term. The latter requirement gives

$$\frac{r_c z_0}{r^2} \ll 1$$

This automatically implies that $z_0 p \sim z_0/r \ll 1$, so in this regime the propagator is independent of z and z' and has the form (63). This means that the interaction between the charges involves integrals $\int dz \rho(z, x)$, so the charge universality holds automatically in the effective four-dimensional theory.

In non-Abelian case, gauge field self-interaction in the bulk occurs with the coupling

$$g_{(5)} = \sqrt{r_c} g_{(4)}$$

With $g_{(4)}$ roughly of order one, the gauge theory in the bulk is strongly coupled at all distances of interest, so the brane-induced mechanism is not suitable for localizing non-Abelian gauge fields.

Translating to gravity, we find that the effective four-dimensional theory obeys the equivalence principle, and the four-dimensional Newton's law between smooth sources is valid at distances

$$r_c \gg r \gg \sqrt{r_c z_0} \quad (64)$$

where

$$r_c = \frac{L_{(5)}^3}{L_{Pl}^2}$$

The interval (64) is large enough if z_0 is sufficiently small. Requiring $r_c > 10$ kpc and $\sqrt{r_c z_0} > 0.1$ mm, we find

$$z_0 < 10^{-26} \text{ cm} \quad (65)$$

There are two problems with this scenario. One is the scalar-tensor structure of the four-dimensional graviton propagator [21]. Another stems from the argument of Ref. [23] that one can trust the calculations leading to eq. (62) only if the transverse distances are larger than $L_{(5)}$. For $r_c \sim 10$ kpc one has $L_{(5)} \sim 10^{-15}$ cm, and from (65) we find that $z_0 \ll L_{(5)}$, in conflict with the latter argument. It remains to be understood how serious these two problems are; one approach to get around at least some of these problems is suggested in Ref. [25].

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